Math 111I
Joseph C Foster
Fall 2017
Exam 2

Name: _____Solutions_____

October 26th, 2017
Time Limit: 75 minutes

This exam contains 10 pages (including this cover page) and 20 questions.
The total number of points is 100. You have 75 minutes to complete the exam.

Read each question carefully. When specified, you must show **all** *necessary* work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	3		11	2	
2	3		12	4	
3	3		13	6	
4	3		14	8	
5	3		15	10	
6	3		16	6	
7	3		17	7	
8	3		18	9	
9	3		19	9	
10	3		20	9	
			Total	100	

1. (3 marks) True or False: There is only one line parallel to $y = 2x + 3$.

     A. True   B. False ⟵ circled

2. (3 marks) True or False: The lines $y = 5x + 2$ and $y = 5(x - 1)$ intersect.

     A. True   B. False ⟵ circled

3. (3 marks) True or False: The average rate of change between two points (x_1, y_1) and (x_2, y_2) is $\dfrac{x_1 - y_1}{x_2 - y_2}$.

     A. True   B. False ⟵ circled

4. (3 marks) Fill in the blank: The lines $y_1 = mx + b$ and $y_2 = nx + c$ are *parallel* if

$$\underline{\qquad\qquad m = n \qquad\qquad}$$

5. (3 marks) Fill in the blank: The lines $y_1 = mx + b$ and $y_2 = nx + c$ are *perpendicular* if

$$\underline{\qquad\qquad mn = -1 \qquad\qquad}$$

6. (3 marks) A balloon is being filled with air. The linear equation

$$V(t) = 5 + 0.5t$$

models the volume V (in cubic feet) of air in the balloon at any time t (in seconds). How many *minutes* will it take until the balloon contains 60ft^3 of hydrogen?

$V(t) = 5 + \frac{1}{2}t = 60$

$\Rightarrow \frac{1}{2}t = 55$

$\Rightarrow t = 110$ seconds

$$\underline{\qquad \dfrac{110}{60} \text{ minutes} \qquad}$$

For questions 7-10, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 1 mark.

7. (3 marks) In an exponential model $P(t) = Ca^t$, what is C?

    A. Initial value

    B. Growth rate

    C. Growth factor

    D. The grade I want on this exam

8. (3 marks) In an exponential model $P(t) = Ca^t$, what is a?

    A. Initial value

    B. Growth rate

    C. Growth factor

    D. The grade I *really* want on this exam

9. (3 marks) In an exponential model $P(t) = Ca^t$, what is the relationship between a and r?

    A. $a = r - 1$

    B. $r = 1 - a$

    C. $r = a - 1$

    D. $a = r$

10. (3 marks) In an exponential model $P(t) = Ca^t$, if t is measured in minutes, what would the equation look like if we instead measured t in hours?

    A. $P(t) = Ca^{\frac{t}{60}}$

    B. $P(t) = Ca^{60t}$

    C. $P(t) = Ca^t$

    D. $P(t) = Ca^{\frac{t}{24}}$

For questions 11 − 21 show all *necessary* work to receive full credit. Circle or box your final answer.

Questions 11 − 15 are the questions written by you.

11. (2 marks) Find the equation of the line with slope 9 and y-intercept 8.

$$y = 9x + 8$$

12. (4 marks) A potato famine is coming to England. One family's farm starts out with 500 potatoes on Sunday. The next day, 450 potatoes are left.

a) Find an exponential model for the amount of potatoes remaining after x days.

$$f(x) = 500\left(\frac{1}{2}\right)^x$$

b) Find an en exponential model for the amount of potatoes remaining after x hours

$$f(x) = 500\left(\frac{1}{2}\right)^{x/24}$$

13. (6 marks) For an accounting job at a company X, the table shows the money you would make based on the number of years experience you have. Find the average rate of change in salaries for each of the given ranges.

Years	Salary	Years	Salary
0	40,000	8	57,000
2	44,000	10	61,000
4	46,000	12	63,000
6	50,000	14	70,000

a) $0 - 2$ years

$$\frac{44,000 - 40,000}{2 - 0}$$

$$= \boxed{2000}$$

b) $8 - 10$ years

$$\frac{61,000 - 57,000}{10 - 8}$$

$$= \boxed{2000}$$

c) $12 - 14$ years

$$\frac{70,000 - 63,000}{14 - 12}$$

$$= \boxed{3500}$$

14. (8 marks) Mr Joe runs his own taxi service for college students at USC. He charges an initial fee of \$5 and then charges 50 cents for every minute in the car.

a) Create a linear model that represents the cost C of a taxi ride lasting t minutes.

$$C(t) = 5 + \tfrac{1}{2}t$$

b) How much would it cost for Mr Joe to take you to Charleston, if Charleston is 2 hours away?

$$C(120) = 5 + \tfrac{1}{2}(120)$$
$$= 5 + 60 = \boxed{\$65}$$

c) Mr Joe's prices compete against Uber's prices. Uber charges an initial fee of \$3 dollars, but charges \$1 per minute in the car. How long would a journey have to take for Mr Joe and Uber to charge the same amount?
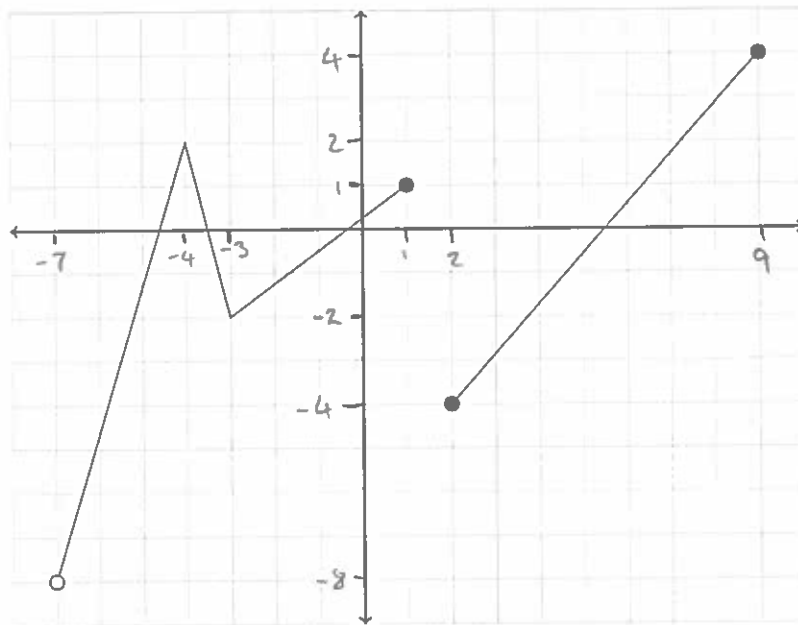
$$U(t) = 3 + t$$

$$U(t) = C(t)$$

$$3 + t = 5 + \tfrac{1}{2}t$$

$$\tfrac{1}{2}t = 2$$

$$\boxed{t = 4 \text{ minutes}}$$

15. (10 marks) Consider the graph below and give the properties listed. You may assume that each square represents 1 unit. (If you think there is more than one answer, just give one of them)

a) What is the domain of this function?

$$(-7, 1] \cup [2, 9]$$

b) What is the range of this function?

$$(-8, 4]$$

c) Where is this function increasing?

$$(-8, -4)$$

d) Where is this function decreasing?

$$(-4, -3)$$

e) Does this function have a local minimum? If so give the coordinates.

$$(-3, -2)$$

f) Does this function have a local maximum? If so give the coordinates.

$$(-4, 2)$$

g) Does this function have a global minimum? If so give the coordinates.

No

h Does this function have a global maximum? If so give the coordinates.

$$(9, 4)$$

16. A graphic artist needs to construct a deign that uses a rectangle whose length is 5cm longer than its width.

   (a) (3 marks) Construct a model P that gives the *perimeter* of the rectangle in terms of its width x.

$$P(x) = x + x + (x+5) + (x+5)$$
$$= 4x + 10$$

   (b) (3 marks) How wide is the rectangle if it has a perimeter of 26cm?

$$4x + 10 = 26$$
$$4x = 16$$
$$\boxed{x = 4_{cm}}$$

17. An architect is designing a building whose footprint has the shape shown.

   (a) (4 marks) Construct a model that gives the total *area* of the footprint of the building.

$$A(x) = 10x + 6x = 16x$$
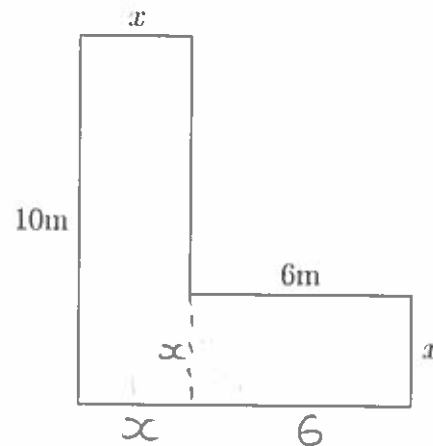
   (b) (3 marks) Find x such that the area of the building is 144m².

$$A(x) = 16x = 144$$
$$\implies \boxed{x = 9}$$

18. Lina is considering installing solar panels on her house. Solar Advantage offers to install solar panels that generate 320kWh of electricity per month for an installation fee of $12,800. She uses 350kWh of electricity per month, and her local utility company charges $0.20 per kWh.

   (a) (3 marks) If Lina gets all her electrical power from the local utility company, find a linear function U that models the cost $U(x)$ of electricity for x months service.

   $$U(x) = 350 \left(\tfrac{1}{5}\right) x = 70x$$

   (b) (3 marks) If Lina has Solar Advantage install solar panels on her roof that generate 320kWh of power per month, find a linear function S that models the cost $S(x)$ of electricity for x months service.

   $$S(x) = 12,800 + (350-320)\left(\tfrac{1}{5}\right)x$$
   $$= 12,800 + 6x$$

   (c) (3 marks) Determine the number of months it would take to reach the break-even point for the installation of Solar Advantages solar panels.

   $$12,800 + 6x = 70x$$
   $$12,800 = 64x$$
   $$\boxed{200 = x}$$

19. Kofi wants to buy a new car, and he has narrowed his choices to two models.

Model A sells for \$12,000, gets 25mi/gal, and costs \$300 a year for insurance
Model B sells for \$15,000, gets 40mi/gal, and costs \$425 a year for insurance

Kofi drives about 25,000 miles per year, and gas costs about \$3 a gallon.

(a) (3 marks) Find a linear function A that models the total cost $A(x)$ of owning Model A for x years.

$$A(x) = \text{Initial} + \left(\text{Insurance} + \frac{\text{miles}}{\text{mpg}} \times \text{cost/gallon} \right) x$$

$$= 12,000 + \left(300 + \frac{25,000}{25} \cdot 3 \right) x$$

$$\boxed{A(x) = 12,000 + 3300\,x}$$

(b) (3 marks) Find a linear function B that models the total cost $B(x)$ of owning Model B for x years.

$$B(x) = 15,000 + \left(425 + \frac{25000}{40} \cdot 3 \right) x$$

$$= 15,000 + \left(425 + \frac{2500}{4} \cdot 3 \right) x$$

$$= 15,000 + \left(425 + \frac{1250}{2} \cdot 3 \right) x$$

$$= 15,000 + \left(425 + 625 \cdot 3 \right) x$$

$$B(x) = 15,000 + (2300) x$$

(c) (3 marks) Find the number of years of ownership for which the cost to Kofi of owning Model A equals the cost of owning Model B.

$$A(x) = B(x)$$

$$12,000 + 3300x = 15,000 + 2300x$$

$$1000x = 3,000$$

$$\boxed{x = 3 \ \text{years}}$$

20. Nuclear power plants produce radioactive plutonium-239, which has a half-life of 20,000 years. An 800 gram sample of plutonium-239 is placed in an underground waste disposal facility.

(a) (2 marks) What is the decay *factor*?

$$\frac{1}{2}$$

(b) (3 marks) Find a function that models the mass $m(t)$ of plutonium-239 remaining in the sample after t years.

$$m(t) = 800 \left(\frac{1}{2}\right)^{t/20,000}$$

(c) (4 marks) How many years will it take for there to be 100 grams of the sample left. (Think about it in half lives and go from there)

$$800 \left(\frac{1}{2}\right)^{t/20,000} = 100$$

$$\left(\frac{1}{2}\right)^{t/20,000} = \frac{1}{8} = \left(\frac{1}{2}\right)^{3}$$

$$\frac{t}{20,000} = 3$$

$$\boxed{t = 60,000 \text{ years}}$$